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Electromagnetic transitions in the ($b\bar{c}$) binding system

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Abstract. We study the electromagnetic decay: $B_c^* \rightarrow B_c e^+ e^-$ with B_c-meson in its ground state B_c-meson in the relativistic independent quark model based on a flavor independent potential in the scalar-vector harmonic form. The transition form factor $F_{B_c^* B_c}(q^2)$ obtained in this model is found to increase linearly with q^2 in the allowed kinematic range of $(2m_e)^2 \leq q^2 \leq (m_{B_c^*} - m_{B_c})^2$. Our predictions for decay width $\Gamma(B_c^* \rightarrow B_c e^+ e^-) = 0.7112 \times 10^{-5}\, KeV$ is compatible with the result of other model calculation based on Bethe-Salpeter approach. The model predictions in this sector would not only yield necessary information about members of B_c family but would provide clue for experimental determination of the unmeasured mass of B_c^* meson which is expected at LHC b and the Z^0 factory in near future.

Keywords: Transition form factor, mass-splitting, decay width, Relativistic independent quark model.

1. Introduction

Ever since its discovery at Fermilab by CDF Collaboration [1] B_c-meson has been attracting a great deal of attention theoretically as well as experimentally. B_c-mesons with explicitly two heavy quarks have not yet been thoroughly studied because of insufficient data available in this sector. The mesons in $b\bar{c}$ (B_c) family lie intermediate in mass and size between charmonium ($c\bar{c}$) and bottomonium ($b\bar{b}$) family. With the observation of B_c at hadron colliders TEVATRON [2] a detailed study of B_c-family members is expected at LHC where the available energy and luminosity are much higher than at TEVATRON that could result in B_c-events thousand times more. Unlike heavy quarkonia Bc-meson with explicitly two heavy quark constituents do not annihilate to photon or gluon. The ground state B_c- meson can therefore decay weakly through $\bar{b} \rightarrow \bar{c}w^{+}$, $c \rightarrow sw^{+}$ or decay radiatively through $\bar{b} \rightarrow \bar{b}\gamma$, $c \rightarrow c\gamma$ at the quark level. $B_c^* \rightarrow B_c e^+ e^-$ is however governed by electromagnetic process in which the emitted photon is an off-shell virtual one. Compared to the radiative decay emitting real photon, the rate of this decay process is thought to be highly suppressed due to a tight three body phase space and an extra electromagnetic vertex. In this process the lepton pair (e^+e^-) could be easily caught by the detectors as clear signals. For description of this decay process, one needs to study the relevant form factor which is the manifestation of non-perturbative QCD process. There have been many theoretical approaches including Light Front Quark model (LFQM), QCD sum rule and Bethe-Saltpeter (BS) approach [3,4] etc. to find the B_c- spectrum, its mass and decay width.

We would like to extend the applicability of relativistic independent quark (RIQ) model to describe this transition. In earlier application of this model, we studied the q^2 dependence of relevant M1- transition [5] and have also reproduced hadronic static properties like hyperfine mass splitting [6], and various decays of hadrons [7] including radiative, weak radiative, rare radiative and radiative leptonic decays of hadrons in the light and heavy flavor sector.

2. Transition Matrix Element, Transition Form factor, and Decay width of RIQ Model

In the RIQ model the decay process, which in fact occurs physically in the momentum eigen state of the participating mesons, a meson state such as $|B_c(P,S_v)>$ is considered at definite momentum $\vec{P}$ and spin state S_v in terms of appropriate wave packet [5,7] as:

$$|B_c(\vec{P},S_V)>=\hat{\Lambda}_{B_c}(\vec{P},S_V)|(\vec{p}_b,\lambda_b);(\vec{p}_c,\lambda_c)> \qquad (1)$$

where $|(\vec{p}_b,\lambda_b);(\vec{p}_c,\ \lambda_c)> \ = \ \hat{b}_b^+(\vec{p}_b,\lambda_b)\,\tilde{b}_c^+(\vec{p}_c,\ \lambda_c)|0>$ is a Fock-space representation of unbound quark (b) and antiquark $(\bar{c})$ in the color singlet configuration with respective momentum and spin as $(\overrightarrow{p_b},\lambda_b)$ and $(\vec{p}_c,\ \lambda_c)$. Here $\hat{b}_b^+(\vec{p}_b,\lambda_b)$ and $\tilde{b}_c^+(\vec{p}_c,\ \lambda_c)$ are respectively the quark and antiquark creation operator and $\hat{\Lambda}$ a bag like integral operator taken in the form

$$\hat{\Lambda}_{B_c}(\vec{P},S_B)=\frac{\sqrt{3}}{\sqrt{N_{B_c}(\vec{P})}}\sum_{\delta_b,\delta_{\bar{c}}}\zeta_{b,\bar{c}}^{B_c}(\lambda_b,\lambda_{\bar{c}})\int d^3\vec{p}_b\,d^3\vec{p}_{\bar{c}}\,\delta^{(3)}(\vec{p}_b+\vec{p}_{\bar{c}}-\vec{P})\mathcal{G}_{B_c}(\vec{p}_b,\vec{p}_{\bar{c}}) \quad (2)$$

Here $\sqrt{3}$ is effective color factor and $\zeta^{B_c}(\lambda_b\lambda_c)$ stands for appropriate spin-flavor coefficient. $N(\vec{P})$ is a meson state normalization factor which is obtained in the integral form

$$N(\vec{P}) = \int d^3\vec{p}_b \left| \mathcal{G}_{Bc}(\vec{p}_b, \vec{P} - p_b) \right|^2 \tag{3}$$

Finally the effective momentum profile function for the quark-antiquark pair in terms of individual momentum probability amplitudes $G_b(\vec{p}_b)$ and $\tilde{G}_c(\vec{p}_c)$ is taken in this model as

$$\mathcal{G}_{B_c}(\vec{p_b}, \vec{p_{\bar{c}}}) = \sqrt{G_b(\vec{p_b})\tilde{G}_{\bar{c}}(\vec{p_{\bar{c}}})} \tag{4}$$

In fact in our earlier works; we have used the individual momentum amplitudes which have been extracted from the momentum projection of the quark orbitals obtained in the model by solving Dirac equation. In the field theoretic description of the decay process such as $B_c^* \rightarrow B_c e^+ e^-$ one calculates the appropriate Feynman diagrams involving free quark and antiquark lines. . Total contribution from Feynman diagrams provides only the constituent level S-matrix element S_{fi}^{bc} which when operated upon by the baglike operator yields meson level effective S-matrix element .

$$S_{fi}^{B_c} = \hat{\Lambda}_{B_c}(\vec{P}, S_B) S_{fi}^{b\bar{c}}$$

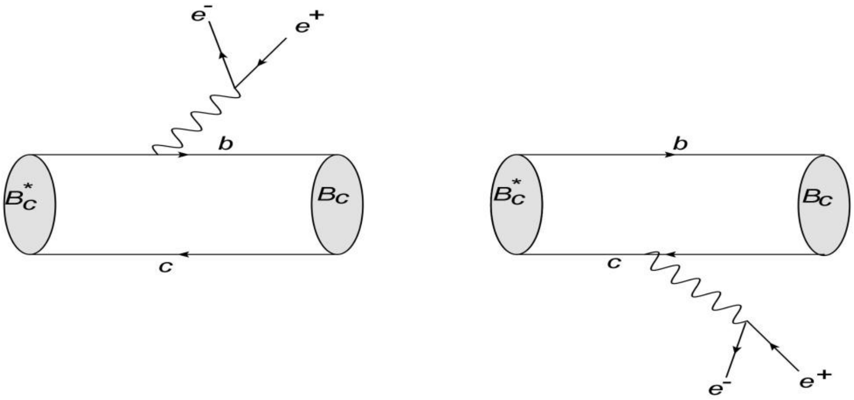

Figure 1: Lowest order Feynman diagram contributing electromagnetic transition

The decay process $B_c^* \rightarrow B_c e^+ e^-$ depicted in figure 1 by the lowest order Feynman diagram is thought to be predominantly a double vertex electromagnetic process governed by photon emission at the photon hadron vertex from independently confined quark as well as antiquark confined in the meson bound state. The emitted photon is an off shell virtual one which ultimately leptonises into a pair of leptons (e^+e^-). Now S_{fi} in B_c^* rest frame is obtained in the standard form as:

$$S_{fi} = (2\pi)^4\delta^{(4)}(k + k_1 + k_2 - \hat{O}M_{B_c^*})\frac{(-i\mathcal{M}_{fi})}{\sqrt{(2\pi)^3 2M_{B_c^*}}}\prod_f \frac{1}{\sqrt{(2\pi)^3 2E_f}} \tag{5}$$

Here k, k_1 and k_2 are four momentum of B_c-meson, electron and positron respectively. On explicit calculation, it is straightforward to show that the time like component of the hadronic matrix element h_0 is zero. With non-vanishing space like component, the invariant matrix element is reduced to the form

$$\mathcal{M}_{fi} = e^2 h_i l^i(k_1, k_2, \delta_1, \delta_2)/(k_1 + k_2)^2 \tag{6}$$

Using usual spin algebra, the non-vanishing space like hadronic part h_i is obtained as

$$h_i = (e_b I_b + e_c I_c)(\vec{\epsilon} \times \vec{k})_i \tag{7}$$

with

$$I_b = \sqrt{2M_{B_c^*}2E_k}\int d\vec{p}_b \frac{\mathcal{G}_{B_c^*}(\vec{p}_b, -\vec{p}_b)\mathcal{G}_{B_c}(\vec{p}_b + \vec{k}, -\vec{p}_b)}{\sqrt{2E_{p_b}2E_{p_b+k}\bar{N}_{B_c^*}(0)\bar{N}_{B_c}(\vec{k})}}\sqrt{\frac{(E_{p_b} + m_b)}{(E_{p_b+k} + m_b)}}$$

$$I_c = \sqrt{2M_{B_c^*}2E_k}\int d\vec{p}_c \frac{\mathcal{G}_{B_c^*}(-\vec{p}_c, \vec{p}_c)\mathcal{G}_{B_c}(-\vec{p}_c, \vec{p}_c + \vec{k})}{\sqrt{2E_{p_c}2E_{p_c+k}\bar{N}_{B_c^*}(0)\bar{N}_{B_c}(\vec{k})}}\sqrt{\frac{(E_{p_c} + m_c)}{(E_{p_c+k} + m_c)}} \tag{8}$$

The decay width can then be calculated from the generic expression:

$$\Gamma = \frac{1}{(2\pi)^5}\frac{1}{2M_{B_c^*}}\int \frac{d\vec{k}d\vec{k}_1 d\vec{k}_2}{2E_{k_1}2E_{k_2}}\delta^{(4)}(k + k_1 + k_2 - \hat{O}M_{B_c^*})\bar{\sum}_{S_V,\delta}|\mathcal{M}_{fi}|^2 \tag{9}$$

in thr form:

$$\Gamma(B_c^* \to B_c e^+ e^-) = \frac{4\alpha_{em}^2}{(2\pi)^3}\int d^3k \bar{\sum}_{S_V,\delta} H_{ij}L^{ij} \tag{10}$$

when,

$$L^{ij} = \int \frac{d\vec{k}_1 d\vec{k}_2}{2E_{k_1}2E_{k_2}}\delta^{(4)}(k + k_1 - \hat{O}M_{B_c^*})\ Tr[(\not{k}_2 + m_2)\gamma^i(\not{k}_1 - m_1)\gamma^j]/(k_1 + k_2)^4 \tag{11}$$

Evaluating trace and adopting standard techniquie of integration via conversion of three momentum integral to four momentum integral, L^{ij} is simplified to

$$L^{ij} = \frac{2\pi}{3}\frac{\delta^{ij}}{(M_{B_c^*-E_K})} \tag{12}$$

Due to δ^{ij} in the expression L^{ij}, H_{ij} is reduced to $H_{ii.}$ Note that a sum over polarization index and spin states and averag over the decaying B_c^* spin states one obtains

$$\bar{\sum}_{S_V,\delta}|(\vec{\epsilon}\times\vec{k})_i|^2 = \frac{2}{3}|\vec{k}|^2 \tag{13}$$

which leads to the contribution of hadronic tensor H_{ii} in terms of transition form factor $F_{B_c^*B_c}$ as

$$\bar{\sum}_{S_V,\delta}H_{ii} = \frac{|\vec{k}|^2}{3}|F_{B_c^*B_c}(q^2)|^2 \tag{14}$$

Now casting the leptonic tensor (12) and hadronic tensor (14) each as function of q^2 and finally integrating out q^2 in the kinematic range: $(2m_e)^2 \le q^2 \le (m_{B_c^*} - m_{B_c})^2$, the decay width is obtained in the form

$$\Gamma(B_c^* \to B_c e^+ e^-) = \frac{2\alpha_{em}^2}{9\pi M_{B_c^*}}\int_{(2m_e)^2}^{(M_{B_c^*}-M_{B_c})^2} dq^2 \frac{E_k(E_k^2 - M_{B_c}^2)^{3/2}}{(M_{B_c^*} - E_k)^2}|F_{B_c^*B_c}(q^2)|^2 \tag{15}$$

where the energy of daughter meson B_c is

$$E_k = \frac{M_{B_c^*}^2 - M_{B_c}^2 - q^2}{2M_{B_c^*}}$$

3. Numerical Results and Discussion

For numerical calculation we take relevant quark masses m_q and corresponding binding energy E_q and potential parameters (a V_0) as those already fixed by fitting the data of heavy flavored mesons [6]. Accordingly we take

$$(a, V_0) \equiv (0.017166\ GeV^3, -0.1375\ GeV)$$

$$(m_b, m_c, E_b, E_c) \equiv (4.77659, 1.49276, 4.76633, 1.57951)\ GeV$$

For meson masses we take our predicted values as M_{B_c}= 6.2642 GeV and $M_{B_c^*}$ = 6.3078 GeV as the mass of B_c^* is unknown. Our predicted M_{B_c},which is obtained by reproducing the hyper fine mass splitting between B_c and B_c^* , is close to the central value (6.2751GeV) of its observed mass.

Now using appropriate wave packets for B_c and B_c^* meson state in the hadronic part and simplifying the leptonic part, the S-matrix element S_{fi} and the invariant transition matrix M_{fi} are calculated from which the model expression for the transition form factor is extracted. We then study the q^2-dependence of $F_{B_c^*B_c}$ in the allowed kinematic range which is depicted in Fig.**2**. We find that the increase of the transition form factor q^2 is linear in the entire kinematic range which is contrary to what is predicted in the work [4] based on Bethe-Salpeter approach where the transition form factor is found almost constant in the entire kinematic range. They considered $F_{B_c^*B_c}(q^2) = F_{B_c^*B_c}(q^2_{min})$ for convenience in their model calculation. On the other hand we have not taken resort to such approximation. Instead we use the form factor as such with the q^2 dependence in the allowed kinematic range in the expression for decay width and then integrate out q^2 which yields our predicted decay width: $\Gamma(B_c^* \rightarrow B_c e^+ e^-) = 0.7112 \times 10^{-5}\ KeV$.

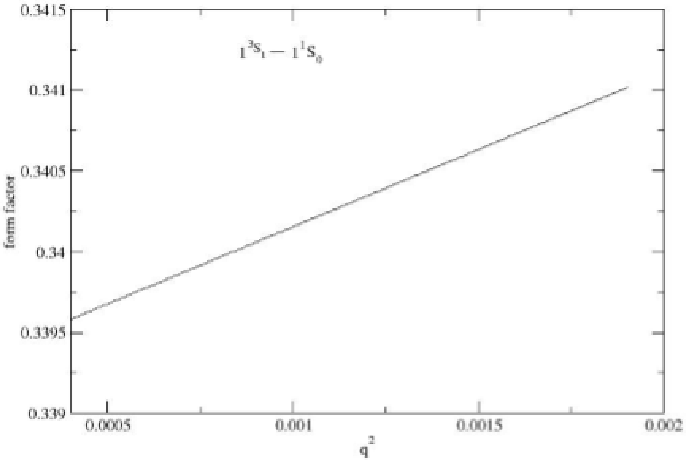

Figure 2: The q^2 dependence of form factor of $B_c^* \rightarrow B_c e^+ e^-$

In the present work the relativistic recoil effect on the antiquark $\bar{c}$ which is not so heavy compared to the quark b is found to be significant. This along with the potential *U(r)* taken in equally mixed scalar-vector harmonic form yields our predicted decay width, which finds an order of magnitude agreement with that of [4], though quantitatively our prediction is different from their result. In the absence of any precise data in this sector one is not sure which model is more suitable to provide a realistic description of the decay process. The theoretical approach to describe this decay mode would provide clue for experimental determination of the unmeasured B_c^* mass. Fortunately the experiments at LHC and Z^0-factory are likely to detect B_c^* meson in near future and provide precise data in this sector.

4. Summary and Conclusion

We study the electromagnetic decay: $B_c^*(1s) \rightarrow B_c(1s)\, e^+e^-$ in the framework of RIQ model based on the interaction potential in the scalar-vector harmonic form. We obtain the momentum probability amplitude for the quark b and antiquark $\bar{c}$ by taking momentum projections of the quark orbitals derivable in this model by solving Dirac equations. For numerical calculation we consider the input parameters: the quark masses m_q, corresponding binding energies E_q and the model parameters (a, V_0) which have been fixed earlier by fitting with the data of the heavy quarkonia. We then predict q^2-dependence of the transition form-factor in the allowed kinematic range $(2m_e)^2 \leq q^2 \leq (m_{B_c^*} - m_{B_c})^2$ and found that the transition form factor $F_{B_c^* B_c}(q^2)$ increase linearly with q^2. Our prediction is in contrast with that of the model calculation based on Bethe-Salpeter approach [4], where they found the form factor to be almost constant in the entire kinematic range. We have avoided such approximation and instead use the form factor as such with its q^2 dependence in the appropriate kinematic range leading to our predicted decay width $\Gamma(B_c^* \rightarrow B_c e^+e^-) = 0.7112 \times 10^{-5}\, KeV$. Incorporating the recoil effect into the analysis, we find the relativistic recoil effect on the antiquark which is not so heavy compared to quark, is found significant. This along with the interaction potential taken in equally mixed scalar-vector harmonic form yields such of our prediction. The future experiments would tell which phenomenological model provides a more realistic description of the decay process. Fortunately, recent advances in experimental probe at LHC and the Z^0-factory are most likely to provide precise data in this sector in near future.

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