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# Hadron-Quark phase transition in low mass neutron stars in a Modified Quark Meson Coupling model

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**Abstract:** The hadron-quark phase transition is studied in the interior of low mass neutron stars. Such a neutron star is considered here as a hybrid star with neutron matter and quark matter. The EOS for the neutron matter has been considered by using a modified quark meson coupling (MQMC) model. In MQMC model we realize the hadrons as a composite of quarks confined by a phenomenological average potential of equal scalar and vector parts. The nucleon nucleon interactions are realized by taking into consideration the exchange of isoscalar-scalar $\sigma$, isoscalar-vector $\omega$ and isovector-vector $\rho$ mesons.To study the phase transition from hadron matter to quark matter we consider the quark matter EOS as derived by Kapusta and have restricted the quark degrees of freedom only to u and d quarks in SU(2) level and with electrons to take care of charge neutrality for hadronic matter. In order to notice the phase transition from hadronic matter to quark matter we have considered the variation of Pressure with chemical potential. It has been observed that at a chemical potential of 1121 MeV and density of 0.25 $fm^{-3}$ there is a phase transition from hadron to quark matter. For neutron-quark hybrid star, TOV equations are solved using the EOS for quark matter and neutron matter. We observe a star mass of 1.46 $M_o$ with a radius of 10.34 km.

**Keywords:** Neutron stars, hadron-quark phase transition, MQMC model.

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## 1. Introduction

Compact stars are the endpoint of life of a star. The decisive factor determining whether a star ends up in white dwarf or neutron star or black hole is the mass of stars. The structure of the neutron star was believed to be made up of only neutron matter but recent studies [1-7] reveal that there might occur a phase transition within the interior of the neutron star.

Quark Hadron phase transition in the core of the neutron star give us an understanding of the structure of the universe i.e, how homogeneous quark matter led to the building up of hadron matter and also the origin of inhomogeneity. The studies of such phase transitions are also helpful in understanding the origin of gravitational waves and the formation of black holes.

In view of this, there are many attempts by different workers to study the occurrence of phase transition in neutron stars. However, in the present work we have made an attempt to study the effects of phase transition from hadron to quark matter in low mass neutron stars. In order to do that first of all the hadron matter EOS is developed in MQMC model [8-10] and the quark matter EOS considered here is due to Kapusta [11]. Then the values of EOS are put in TOV equations and are solved to get the star mass and star radii.

In section II we describe the model for hadronic matter, i.e. Modified quark Meson Coupling model (MQMC). In section III, the EOS for hadronic matter in this model is found. In section IV, the Quark matter EOS is considered as given in [11] and the conditions for phase transition are discussed. In section V, we discuss the results of the present work.

## II. Modified Quark Meson Coupling Model

In the MQMC model the free independent quarks inside the nucleon is considered to be bound by a phenomenological average potential of equal mixture of scalar and vector part.

$$U(r) = \frac{1}{2}(1+\gamma^0)V(r)$$

with

$$V(r) = (ar^2 + V_0), \quad a > 0. \tag{1}$$

The importance of choosing such a potential is that it converts the independent quark Dirac equation into an effective Schrodinger like equation for the upper component of the Dirac spinor which can be easily solved. The potential parameter (a, $V_0$) are determined by fixing the nucleon mass and the charge radius of the proton in free space. In this model the nucleon nucleon interactions are realized by introducing quarks coupling to mesons through mean field approximations.

The confining interaction here provides the zeroth order quark dynamics of the hadron. In the medium, the quark field $\psi_q(\mathbf{r})$ satisfies the Dirac equation

$$\left[\gamma^0\left(\varepsilon_q - V_\omega - \frac{1}{2}\tau_{3q}V_\rho\right) - \vec{\gamma}\bullet\vec{p} - \left(m_q - V_\sigma\right) - U(r)\right]\psi_q(\vec{r}) = 0 \qquad (2)$$

where, $V_\sigma = g_\sigma^q \rho_0$, $V_\omega = g_\omega^q \omega_0$, $V_\rho = g_\rho^q b_{03}$; with $\sigma_0$, $\omega_0$ and $b_{03}$ being the classical meson fields, $g_\sigma{}^q$, $g_\omega{}^q$ and $g_\rho{}^q$ are the quarks coupling to the $\sigma$, $\omega$ and $\rho$ mesons respectively. $m_q$ is the quark mass and $\tau_{3q}$ is the third component of the Pauli matrices. In the present paper, we consider nonstrange $q = u$ and $d$ quarks only. We can now define

$$\epsilon'_q = (\epsilon^*_q - \tfrac{V_0}{2}), \quad m'_q = (m^*_q + \tfrac{V_0}{2}), \qquad (3)$$

where the effective quark energy $\epsilon^*_q = \epsilon_q - V_\omega - \frac{1}{2}\tau_{3q}V_\rho$, and effective quark mass $m^*_q = m_q - V_\sigma$.

We now introduce $\lambda_q$ and $r_{0q}$ as

$$\epsilon'_q + m'_q = \lambda_q \text{ and } r_{0q} = (a\lambda_q)^{-1/4} \qquad (4)$$

The ground state quark energy can be obtained from the eigenvalue condition

$$\epsilon'_q - m'_q\sqrt{\frac{\lambda_q}{a}} = 3 \qquad (5)$$

The solution of equation (5) for the quark energy $\epsilon^*_q$ immediately leads to the mass of the nucleon in the medium in zeroth order as

$$E_N^{*0} = \sum_q \epsilon^*_q \qquad (6)$$

The motion of the spurious centre of mass, the one gluon exchange at short distances and the quark pion coupling arising out of chiral symmetry breaking are eliminated by incorporating appropriate correction terms.

The centre of mass correction $\epsilon_{cm}$ and the pionic corrections $\delta M_N^\pi$ in the present model are found respectively as [7]

$$\epsilon_{cm} = \frac{77\epsilon'_u + 31m'_u}{3(3\epsilon'_u + m'_u)^2 r_{0u}^2} \qquad (7)$$

and

$$\delta M_N^\pi = -\frac{171}{25} I_\pi f_{NN\pi}^2 \qquad (8)$$

Here,

$$I_\pi = \frac{1}{\pi m_\pi^2}\int_0^\infty dk \frac{k^4 u^2(k)}{\omega_k^2} \qquad (9)$$

with the axial vector nucleon form factor given as

$$u(k) = [1 - \frac{3}{2}\frac{k^2}{\lambda_q(5\epsilon_q' + 7m_q')}]e^{-k^2 r_0^2/4} \,. \tag{10}$$

The pseudovector nucleon pion coupling constant $f_{NN\pi}$ can be obtained from the familiar Goldberg Triemann relation using the axial vector coupling constant value $g_A$ in the model.

The color electric and color magnetic contribution to the gluonic correction which arises due to one gluon exchange at short distances are given as:

$$(\Delta E_N)_g^E = \alpha_c(b_{uu} I_{uu}^E + b_{us} I_{us}^E + b_{ss} I_{ss}^E) \tag{11}$$

and due to color magnetic contributions, as

$$\Delta E_{N_g}^{\ M} = \alpha_c(a_{uu} I_{uu}^M + a_{us} I_{us}^M + a_{ss} I_{ss}^M \,, \tag{12}$$

where $a_{ij}$ and $b_{ij}$ are the numerical coefficients depending on each baryon. The color electric contributions to the correction of baryon masses due to one gluon exchange are calculated in a field theoretic manner. It can be found that the numerical coefficient for color electric contributions such as $b_{uu}$,$b_{us}$ and $b_{ss}$ comes out zero. From calculations we have $a_{uu} = -3$ and $a_{us} = a_{ss} = b_{uu} = b_{us} = b_{ss} = 0$ for the nucleons. The quantities $I_{ij}^{E,M}$ are given by the following equations

$$I_{ij}^E = \frac{16}{3\sqrt{\pi}}\frac{1}{R_{ij}}[1 - \frac{\alpha_i + \alpha_j}{R_{ij}^2} + \frac{3\alpha_i\alpha_j}{R_{ij}^4}]$$

$$I_{ij}^M = \frac{256}{9\sqrt{\pi}}\frac{1}{R_{ij}{}^3}(\frac{1}{3\epsilon_i' + m_i'})(\frac{1}{\epsilon_j' + m_j'}) \tag{13}$$

Where

$$R_{ij}^2 = 3[\frac{1}{\epsilon_i'^2 - m_i'^2} + \frac{1}{\epsilon_j'^2 + m_j'^2}] \quad \alpha_i = \frac{1}{(\epsilon_i' + m_i')(3\epsilon_i' + m_i')} \tag{14}$$

In the calculation we have taken $\alpha_c = 0.58$ as the strong coupling constant in QCD at the nucleon scale. The color electric contribution is zero here, and the gluonic corrections to the mass of the nucleon are due to color magnetic contributions only.

Finally treating all these corrections independently, the mass of the nucleon in the medium becomes

$$M_N^* = E_N^{*0} - \epsilon_{cm} + \delta M_N^\pi + (\Delta E_N)_g^E + (\Delta E_N)_g^M \tag{15}$$

## III. The Nuclear Matter Equation of State

The total energy density and pressure at a particular baryon density for the nuclear matter becomes

$$\boldsymbol{\varepsilon = \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_{03}^2 + \frac{\gamma}{2\pi^2} \sum_{N=p,n} \int_0^{k_N} k^2 dk \sqrt{k^2 + {M_N^*}^2} + \sum_l \frac{1}{\pi^2} \int_0^{k_l} k^2 dk \sqrt{k^2 + {m_l}^2}} \quad (16)$$

$$\boldsymbol{P = -\frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_{03}^2 + \frac{\gamma}{6\pi^2} \sum_{N=p,n} \int_0^{k_f} \frac{k^4 dk}{\sqrt{k^2 + {M_N^*}^2}} + \sum_l \frac{1}{3\pi^2} \int_0^{k_l} \frac{k^4 dk}{\sqrt{k^2 + {m_l}^2}}} \quad (17)$$

where $\gamma = 2$ is the spin degeneracy factor for nuclear matter. The vector mean-fields $\omega_0$ and $b_{03}$ are determined through

$$\omega_0 = \frac{g_\omega}{m_\omega^2} \rho_B, \qquad b_{03} = \frac{g_\rho}{2 m_\rho^2} \rho_3 \quad (18)$$

where $g_\omega = 3 g_\omega^q$ and $g_\rho = g_\rho^q$. Finally, the scalar mean-field $\sigma_0$ is fixed by

$$\frac{\partial \varepsilon}{\partial \sigma_0} = 0 \quad (19)$$

The iso-scalar scalar and iso-scalar vector couplings $g_\sigma^q$ and $g_\omega$ are fitted to the saturation density and binding energy for nuclear matter. The isovector vector coupling $g_\rho$ is set by fixing the symmetry energy. For a given baryon density, $\omega_0$, $b_{03}$ and $\sigma_0$ are calculated from the equation (18) and (19) respectively. The chemical potentials, necessary to define the $\beta-$ equilibrium conditions, are given by

$$\mu_N = \sqrt{k_N^2 + M_N^{*2}} + g_\omega \omega_0 + g_\rho \tau_{3N} b_{03} \quad (20)$$

where $\tau_{3N}$ is the isospin projection of the nucleon N.

The lepton Fermi momenta are the positive real solutions of $(k_e^2 + m_e^2)^{1/2} = \mu_e$ and $(k_\mu^2 + m_\mu^2)^{1/2} = \mu_\mu$. The equilibrium composition of the star is obtained by solving the equations of motion of meson fields in conjunction with the charge neutrality condition, given as

$$q_{tot} = \sum_N q_N \frac{\gamma k_N^3}{6\pi^2} + \sum_{l=e,\mu} q_l \frac{k_l^3}{3\pi^2} = 0 \quad (21)$$

where $q_N$ corresponds to the electric charge of nucleon species $N$ and $q_l$ corresponds to the electric charge of lepton species $l$. The total density is given by $\rho = \sum_N \gamma k_N{}^3 / \left(6\pi^2\right)$.

Following the determination of the EOS the relation between the mass and radius of a star with its central density can be obtained by integrating the Tolman-Oppenheimer-Volkoff (TOV) [12] equations given as,

$$\frac{dP}{dr} = -\frac{G}{r}\frac{[\varepsilon+P][M+4\pi r^3 P]}{(r-2GM)} \quad (22)$$

$$\frac{dM}{dr} = 4\pi r^3 \varepsilon \quad (23)$$

with $G$ as the gravitational constant and $M(r)$ as the enclosed gravitational mass. We have used $c$ = 1.

The equations are then integrated from the origin as an initial value problem for a given choice of the central energy density, ($\varepsilon_0$). Of particular importance is the maximum mass obtained from the solution of the TOV equations. The total radius r = R is obtained from the condition where pressure vanishes.

**Table 1:** Parameters for nuclear matter. They are determined from the binding energy per nucleon, $B.E = B_0 \equiv E/\rho_B - M_N = -15.7$ MeV and pressure, $P = 0$ at saturation density $\rho_B = \rho_0 = 0.15$ fm$^{-3}$.

| $m_q$ (MeV) | $g_\sigma^q$ | $g_\omega$ | $g_\rho$ | $a$(fm$^{-3}$) | $V_0$(MeV) |
|---|---|---|---|---|---|
| 30 | 5.4676 | 3.9697 | 8.9903 | 0.91 | 103.78 |

## IV. Quark Matter Equation of state and phase transition

In dense stars, due to high pressure at the core, the hadronic matter are believed to form a deconfined quark state. In terms of baryon and electron chemical potentials $\mu_B$ and $\mu_E$, we have

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_E\ , \mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_E\ , \mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_E \quad (24)$$

using quark counting argument.

The pressure contributed by the quarks is computed to the order $\alpha_s = \frac{g^2}{4\pi}$ where g is the QCD coupling constant. Confinement is simulated by a bag constant B.
The electron pressure is

$$P_e = \frac{\mu_e^4}{12\pi^2} \tag{25}$$

The pressure for quark flavour f, with f = u,d or s is

$$P_f = \frac{1}{4\pi^2}\left[\mu_f k_f\left(\mu_f^2 - 2.5m_f^2\right) + 1.5m_f^4 \ln\left(\frac{\mu_f+k_f}{m_f}\right)\right] - \frac{\alpha_s}{\pi^2}[\frac{3}{2}\left(\mu_f k_f - m_f^2 \ln\left(\frac{\mu_f+k_f}{m_f}\right)\right)^2 - k_f^4] \tag{26}$$

The fermi momentum is $k_f = (\mu_f^2 - m_f^2)^{1/2}$ (27)

The total pressure including the bag constant B is :

$$P = P_e + \sum_f P_f - B \tag{28}$$

$\mu_B$ and $\mu_E$ are two independent chemical potentials. $\mu_E$ is adjusted so that the matter is electrically neutral i.e. $\frac{\partial P}{\partial \mu_E} = 0$. The baryon number density is given by $\rho =. \frac{\partial P}{\partial \mu_B}$.

We use Gibbs' criteria to study the phase transition from cold neutron matter to quark matter and to determine the region of co-existence of neutron and quark phase.

The critical pressure and the critical potential are determined from the condition

$$P_{nm}(\mu_B) = P_{qm}(\mu_B) \tag{29}$$

## V. Results and Discussion

We set the relativistic quark model parameters $(a, V_0)$ by fitting the nucleon mass $M_N$ = 939 MeV and charge radius of the proton $\langle r_N \rangle$ = 0.87 fm in the free space. Taking standard values for the meson masses, namely $m_\sigma$ = 550 MeV, $m_\omega$ = 783 MeV and $m_\rho$ = 763 MeV and fitting the quark-meson coupling constants self consistently, we obtain the correct saturation properties of nuclear matter binding energy, $B.E. \equiv B_0 = E/\rho_B - M_N = -15.7$ MeV, pressure, $P$ = 0 and symmetry energy $J$ = 32.0 MeV at $\rho_B = \rho_0 = 0.15$ fm$^{-3}$. The values of $g_\sigma^q, g_\omega$ and

$g_\rho$ obtained in the model with the quark mass fixed at 30 MeV are given in Table 1. The EOS for the nuclear matter consisting of neutrons and protons are obtained from the relativistic quark model with the above set of parameters using equations (16) and (17). For the quark matter EOS, the Bag constant is fixed at B = $(174\ \text{MeV})^4$.

In Fig. 1, we plot the pressure P versus chemical potential $\mu_B$ for neutron matter and quark matter. It is seen from the (P, $\mu_B$) curves that the critical values of Pressure and Chemical Potential are 0.25fm$^{-3}$ and 1121 MeV respectively. The corresponding critical energy densities for neutron matter and quark matter are given as $\epsilon_{cric} = 2.55$ fm$^{-4}$ and $\epsilon_{cric} =$ 3.39 fm$^{-4}$. The corresponding baryon number densities for neutron matter and quark matter are $\rho_{cric} =$ 0.510 fm$^{-3}$ and $\rho_{cric} =$ 0.643 fm$^{-3}$. It is seen that there occurs a phase transition from hadron to quark matter at around 3.4 times the nuclear matter density. The order of the phase transition will be studied in detail in our future work. In Fig. 2 we plot the mass radius curve. From the graph it is seen that the mass of the star is coming out to be 1.46 $M_\circ$ and the star radius is 10.34 km.

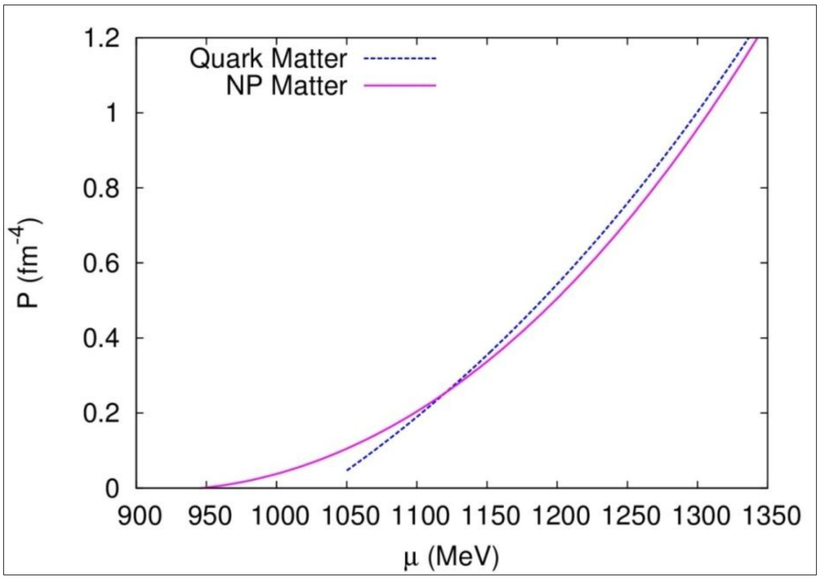


**Figure 1:** Pressure as a function of chemical potential for nuclear matter and quark matter at quark mass $m_q = 30$ MeV.

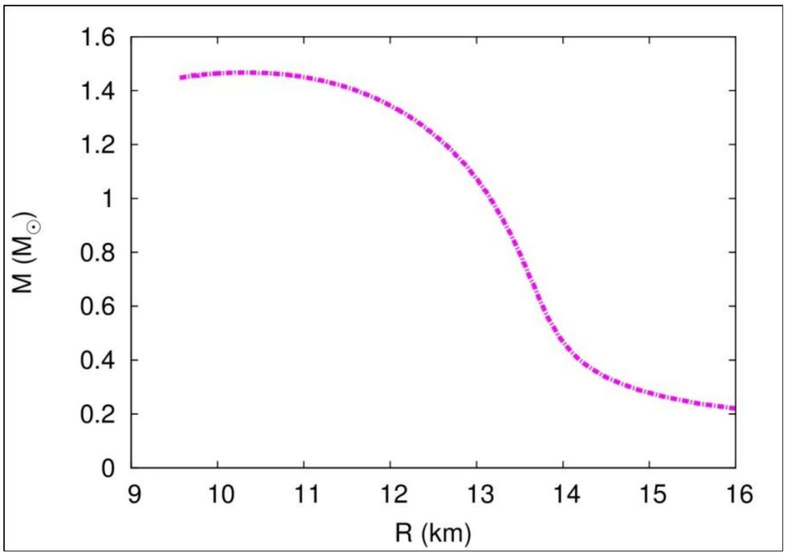


**Figure 2:** Star mass as a function of radius at quark masses $m_q$ = 30 MeV.

## VI. Conclusion

In the present work the phase transition from hadronic matter to quark matter in low mass neutron stars is studied. The EOS of hadronic matter has been developed in MQMC model by taking the spurious centre of mass correction, pionic correction for restoration of chiral symmetry and gluonic corrections to take care of the short range quark-quark interactions inside the hadron. The EOS for the quark matter has been taken from Kapusta [11]. They are then fed into the TOV equations to calculate the star mass and radii. We observe that the hadronic matter in the neutron stars undergoes a phase change to quark matter around the critical baryon density of 0.510 $fm^{-3}$ which is around 3.4 times nuclear matter saturation density. Upon solving the TOV equations we find that the mass of the star is1.46 $M_o$ and the corresponding radius is 10.34 km. Such star can be taken as a low mass neutron star.

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